UNIVERSITY OF PITTSBURGH
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OF BUSINESS

Financial Management I

Workshop on Time Value of Money

MBA

2016 – 2017

Finance & Valuation

- Capital Budgeting Decisions
  - Long-term Investment decisions
  - Investments in Net Working Capital
- Financing Decisions
  - Raising funds
  - Capital structure policy
- Corporate Governance Structure
  - Ownership, control, incentives
- Risk Management
  - Managing the firm's exposures

Financial Statements

- Study Chapter 2 prior to class!
- Read Chapter 3.1 and 3.2!
- Balance sheet (stock)
    - Liquidity of Assets
    - Debt versus Equity
    - Value versus Cost
        - Carrying or Book Value
        - GAAP dictates carrying assets at cost!
        - Finance is about determining value!
- Income Statement (flow)
    - Revenue – Expenses = Income
- Cash Flow – defined in FM-1

Valuation

- *Value = f(Asset Characteristics)*
- Asset Characteristics

1. What is the cash flow generating capacity?

   CASH FLOW

2. What is the time horizon of the cash flows?

   TIME VALUE OF MONEY

3. What is the risk associated with the cash flows?

   RISK

Time Value of Money: Roadmap

- Defining NPV in single period setting
- Generalize to multiple periods
- Simplifications/Shortcuts
  - Perpetuities & Growing Perpetuities
  - Annuities & Growing Annuities
- Interest rate conversions
- Inflation
- Practice Examples

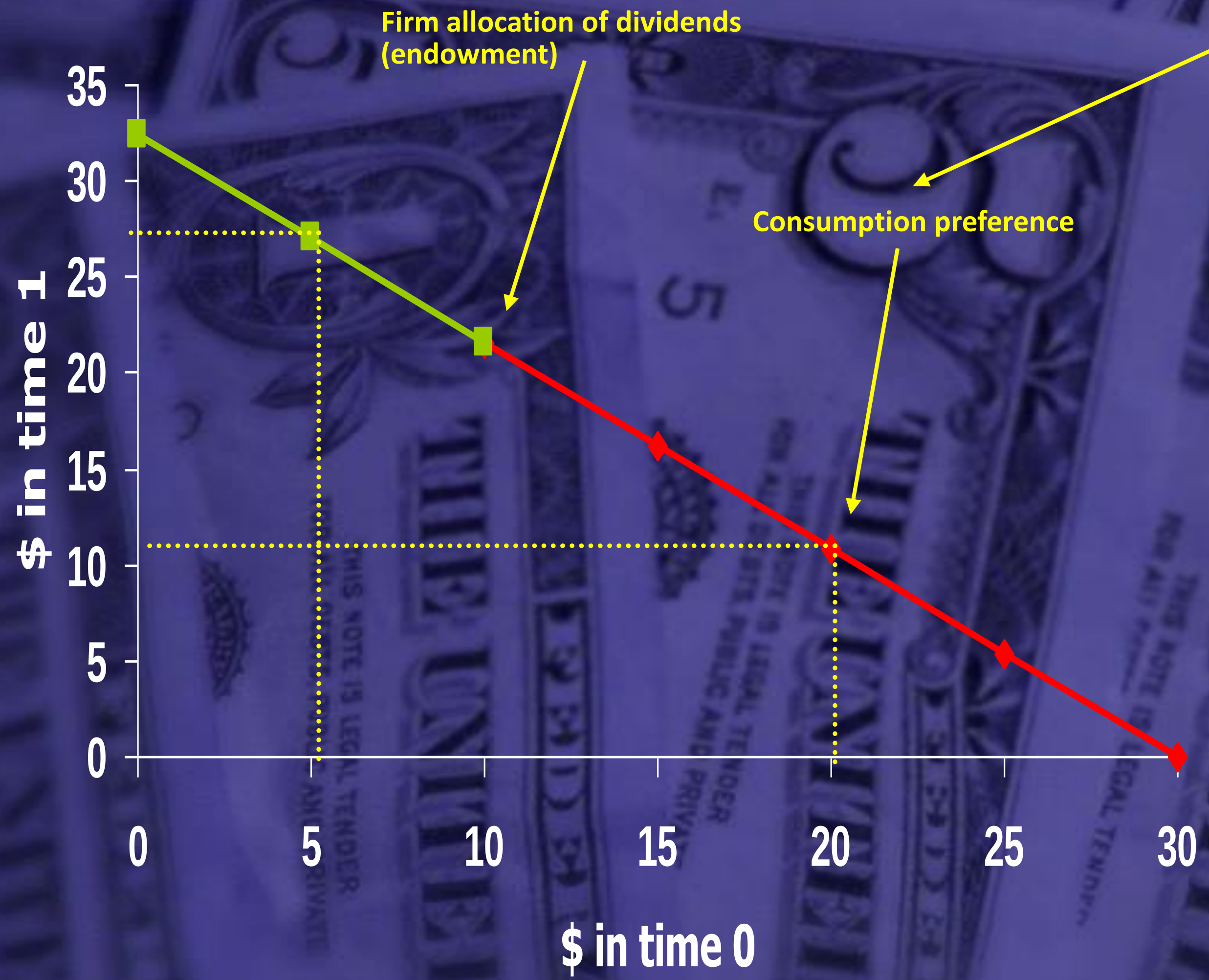
Suggested Practice Problems

- ***Practice Problems (11th edition):***
  - *Chapter 4: 7, 9, 12, 13, 17, 27, 28, 32, 53, 56*
- ***Practice Problems (9th or 10th edition):***
  - *Chapter 4: 13, 14, 15, 19, 33, 38, 52, 54, 68*

Valuing Projects: Understanding NPV

- NPV = Net Present Value = Present Value of Expected Cash Inflows minus Present Value of Expected Cash Outflows
- Assume (for now) no uncertainty and one time period (beginning to end, i.e., from t=0 to t=1)
- Mr. T owns 1 share in a firm and is rational . . . .
- Cash flow/share for the firm is as follows:
    - $CFPS_0$ = \$10 and $CFPS_1$ = \$21.60
- Firm pays all cash flows as dividends and liquidates at t=1
    - DPS_0 = \$10 and DPS_1= \$21.60
- Borrowing and lending rate is 8% in capital market
- No agency problem, no asymmetric information

The Opportunity Set with Borrowing and Lending



- **Consume $20 at t = 0**
  - **Borrow $10**
  - **Payback $10.80**
  - **C1 = 21.6-10.8 = $10.80**
- Consume $30
  - Borrow $20
  - Payback $21.60
  - C1 = 21.6-21.6 = $0
- Consume $5
  - Lend $5
  - Return $5.40
  - C1 = =21.6+5.4 = $27
- Consume $0
  - Lend $10
  - Return $10.80
  - C1 = 21.6+10.8 = $32.40

What if the firm has two Investment Opportunities?

- Cash flows based on past investments
  - $CFPS_0$ = 10, $CFPS_1$ = 21.60
- Investment opportunities:

1. Investment $5 per share @ 62% (ROIC)
2. Investment $5 per share @ 5% (ROIC)

- What should the firm do?

Consider Investment 1

- DPS_0 = \$5
- Borrow additional \$15 to maintain consumption preference
- Return on investment
  - \$5 × 1.62 = \$8.10
- $CFPS_1$ = 21.60 + 8.10 or \$29.70 = D_1
- Pay off loan:
  - 15 × 1.08 = 16.20
- Consumption in year 1:
  - C_1 = 29.70 – 16.20 = \$13.50 > \$10.80 !!! So, \$2.50 is created in value!!!

- Old (current) wealth
  - W_0=10 + (21.6 / 1.08)=30
- New (current) wealth
  - W_0^*=5 + (29.7 / 1.08)=32.5
- NPV of investment: NOTICE how it is expressed in *changes*!!!
  - -5 + (8.10 / 1.08)= 2.5

Consider Investment 2

- DPS_0 = \$0
- Return on investment
  - \$5 × 1.05 = \$5.25
  - $CFPS_1$ = 29.70 + 5.25 or \$34.95 = D_1
- Old wealth
  - W_0= 5 + (29.7 / 1.08) = 32.5
- New wealth
  - W_0^*=34.95 / 1.08 = 32.36
- NPV of investment
  - –5 + (5.25 / 1.08) = –0.14
- NPV is additive!
- How about flexibility?

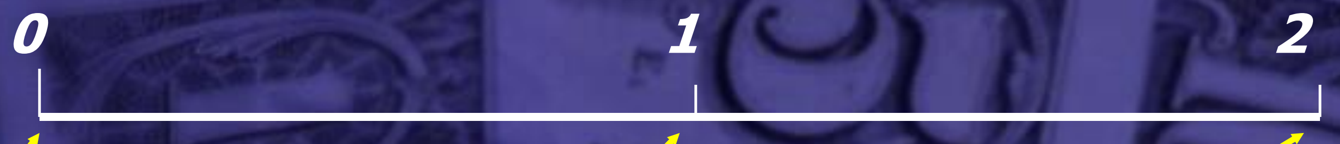
NPV as a decision tool

- NPV of investment = Δ in shareholder current wealth as a result of the investment:

$$NPV = -Cost + \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$$

- Decision rule if management maximizes current shareholder wealth:
  - Invest as long as NPV > 0
  - Invest as long as the rate of return (on margin) is larger than the discount rate (ROIC > r → NPV > 0)
- Investment decision supported by shareholders regardless of their consumption preferences

Extending to multiple periods



C_0

$FV_1 = C_0(1+r)$

$FV_2 = FV_1(1+r)$
$= C_0(1 + r)^2$

In general, $FV_T = C_0(1 + r)^t$

How much would $200,000 be worth in 25 years @ 8%?

$FV_{25} = 200{,}000(1.08)^{25} = \$1{,}369{,}695$

Present Values and Multiple Periods

0 1 2

$PV = PV_1/(1 + r)$
$\quad = C_2/(1 + r)^2$

$PV_1 = C_2/(1 + r)$

C_2

$$PV = C_t / (1 + r)^t$$

Note that this is the same formula as for FV

What is the maximum price you would pay today for a machine that generates a single cash flow of \$2,000,000 in 20 years? Interest rate is 8%

What if you sell this machine?

$PV = 2{,}000{,}000/(1.08)^{20} = \$429{,}096$

Multiple Cash Flows

- C_t in year t, cash flows last for *t* years
  - $PV = [C_1/(1+r)] + [C_2/(1+r)^2] + \ldots\ldots + [C_T/(1+r)^T]$
  - $PV = \Sigma C_t / (1+r)^t$

Finding the Number of Periods or solving for "*r*"

- Sometimes we will be interested in knowing how long it will take our investment to earn some future value. Given the relationship between present values and futures value, we can also find the number of periods. We can solve for the number of periods by rearranging the following equation:

$FV = PV \times (1 + r)^t \rightarrow FV / PV = (1 + r)^t \rightarrow \ln(FV / PV) = \ln (1 + r)^t$

$\rightarrow \ln(FV) - \ln(PV) = t \times \ln (1 + r)$

$\rightarrow t = (\ln(FV) - \ln (PV)) / \ln (1 + r)$

- How long would it take to double your money at 5%?
- Answer: Approximately 14 years and 2 months
- What yearly interest rate are you offered if your bank promises $800 three years from now, when you make a $750 deposit?
- Answer: $800 = 750 \times (1 + r)^3 \rightarrow (800/750)^{1/3} = 1 + r \rightarrow r = 2.175\%$

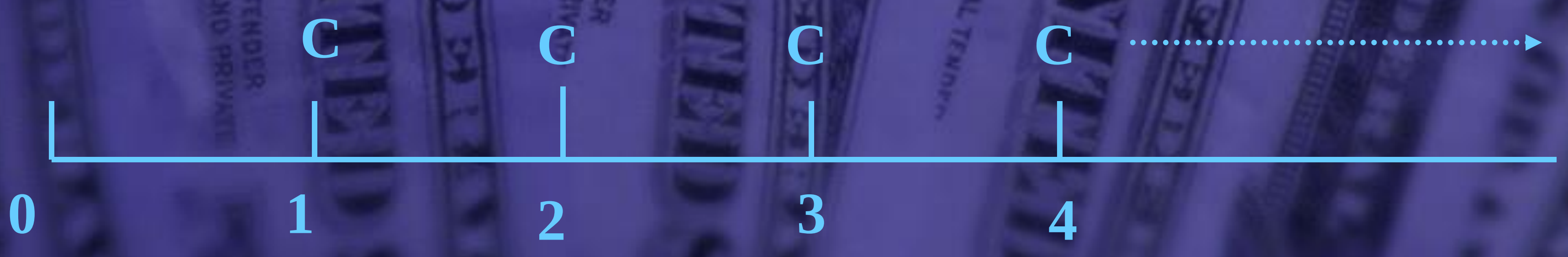
Multiple and Infinite Cash Flows

- Annuity: Finite stream of identical cash flows
- Perpetuity: Infinite stream of identical cash flows
- Identical: separated by an identical growth rate (g=0 in this example)

Short Cuts

Perpetuity - Investment in which a cash flow is theoretically received forever.

$$PV = \frac{C}{r}$$



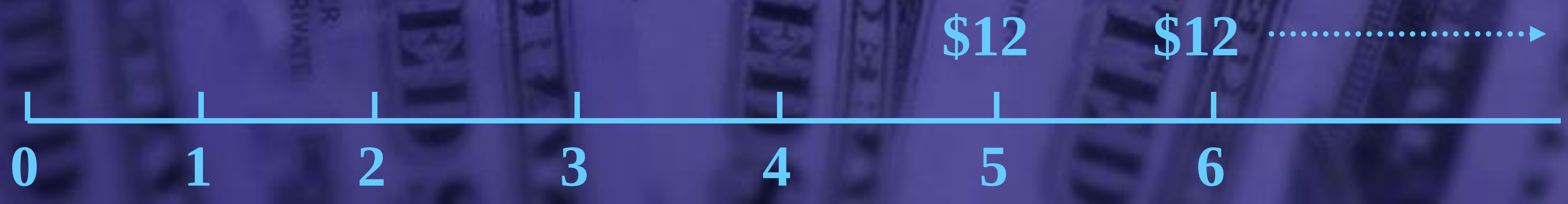
Perpetuities: Examples

- Consol that pays \$100 per year forever, interest rates are at 8% ($C = 100$, $r = 8\%$)

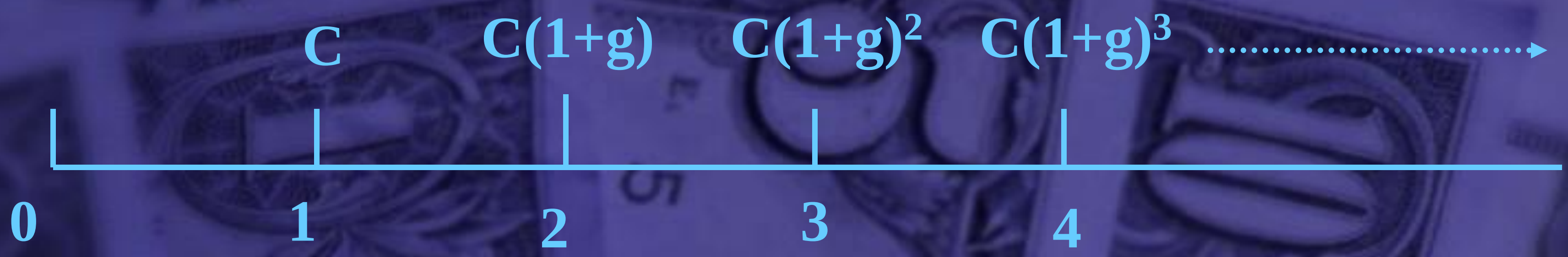
  $PV = 100/0.08 = \$1,250$

- Security that is expected to pay \$12 starting in 5 years, payments will remain constant and last forever, interest rates are at 8%

  $C = 12$, $r = 8\%$
  $PV_4 = 12/0.08 = \$150$
  $PV = PV_4/(1+r)^4 = 150/(1.08)^4 = \$110.25$



Multiple Cash Flows
A Growing Perpetuity



- $C_1 = C$, cash flow grows by g% every year, cash flows last forever: $g < r$

  $PV = [C/(1+r)] + [C(1+g)/(1+r)^2] + \ldots\ldots\ldots\ldots\ldots\ldots$
  $+ [C(1+g)^{(T-1)}/(1+r)^T] + \ldots\ldots$

  $PV = C/(r-g)$

Growing Perpetuities: Examples

- **Security that will pay \$100 next year, payments will grow at 6% per year, interest rates are at 8% (C = 100, r = 8%, g = 6%)**

  $PV = C/(r - g) = 100/(0.08 - 0.06) = \$5{,}000$

  Value of growth = 5000 – 1250 = \$3,750

- **Security that is expected to pay \$12 starting in 5 years, payments will grow at 2% per year and last forever, interest rates are at 8%**

  C = 12, r = 8%, g = 2%

  $PV_4 = C/(r - g) = 12/(0.08 - 0.02) = \$200$

  $PV = PV_4/(1+r)^4 = 200/(1.08)^4 = \$147$

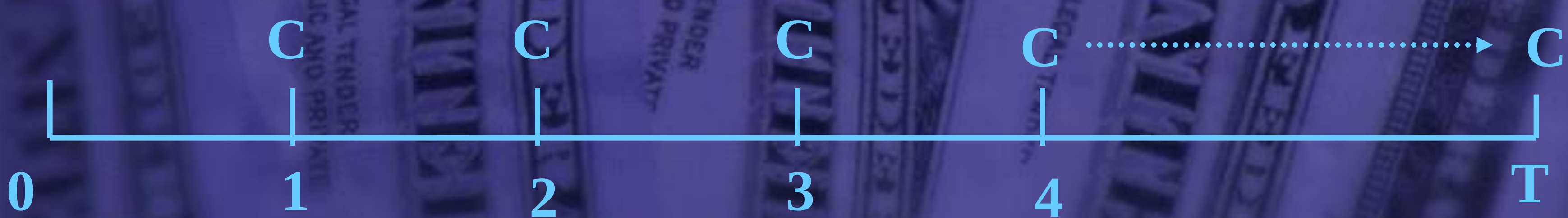
  Value of growth = 147 – 110.25 = \$36.75

Short Cuts

Annuity - An asset that pays a fixed sum each year for a specified number of years.

$$\text{PV of annuity} = \frac{C}{r} \times \left[1 - \frac{1}{(1+r)^t} \right]$$

$$FV \text{ of annuity} = \frac{C}{r}\left[(1+r)^t - 1\right]$$



Annuity Short Cut

Example

You agree to lease a car for 4 years at $300 per month. You are not required to pay any money up front or at the end of your agreement. If your opportunity cost of capital is 0.5% per month, what is the cost of the lease?

$$\text{Lease Cost} = \frac{300}{.005} \times \left[1 - (1 + .005)^{-48}\right]$$

$$Cost = \$12{,}774.10$$

Examples of Annuities

- Buy a car with a \$40,000 loan for 48 months at 0.85%
- What is the monthly payment?
  - $PV = 40{,}000 = [C/0.0085][1 - (1.0085)^{-48}]$
  - $40{,}000 = 39.28C$
  - $C = \$1{,}018.35$

Examples of Annuities

- You buy a security that promises 10 payments of \$1,000 every three years.
- What should you be willing to pay for this security if your opportunity cost is 10% per year?

C = 1,000, t = 10, r = ?

Converting interest rates

- If annual interest rate is 10%, then how much would the three-year rate be?
  - If you invest \$100, at the end of the year you would have \$110
  - At the end of the second year you would have \$121 = $(1.1)^2$
    - The two-year interest rate would be 21%
  - At the end of the third year you would have \$133.1 = $(1.1)^3$
    - The three-year interest rate would be 33.1% (3-YEAR EFFECTIVE RATE)
- PV = $[1000/0.331][1 - (1.331)^{-10}]$ = \$2,848

Examples of Annuities

- You are planning to save for the next 10 years for your son's college education. You estimate it will take him four years to finish school at a cost of $30,000 per year starting 11 years from now. How much would you have to save each year, starting one year from now, if you expect your money to earn 8% per year?
- Either FV or PV solution works!
- PV in year in year 10 of future education costs:
  $[30{,}000/0.08][1-(1.08)^{-4}] = \$99{,}364$
- PV in year 0 of these costs:
  $\$99{,}364/(1.08)^{10} = \$46{,}025$
- PV of saving \$C per year for 10 years
  $\$46{,}025 = [C/0.08][1 - (1.08)^{-10}] \rightarrow C =$ **$6,859**

Growing Cash Flows

Growing Perpetuity

	C	C(1+g)	C(1+g)^2	C(1+g)^3	…
0	1	2	3	4	

Growing Annuity

- **Growing Perpetuity:**

$$PV = \frac{C}{r-g}$$

Growing Annuity:

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^t\right] \text{ if } r \neq g$$

$$PV = \frac{t \times C}{1+r} \text{ if } r = g$$

What about a (growing) perpetuity starting in year 4?

2 types of Time Value Formulae:

I: A single cash flow moved multiple time periods

II: Multiple cash flows moved a single time period

Example of a Growing Annuity

- You want to buy a house with a 30-year mortgage, The first payment is \$2,500. Payments will grow at 0.2%/month and the interest rate is 0.67%(C = 2,500; r = 0.67; t = 360; g = 0.2)
- What is the maximum you can borrow if you assume that your first payment will be one month after you close on the mortgage?
- $PV = [C/(r-g)][1 - \{(1+g)/(1+r)\}^t]$

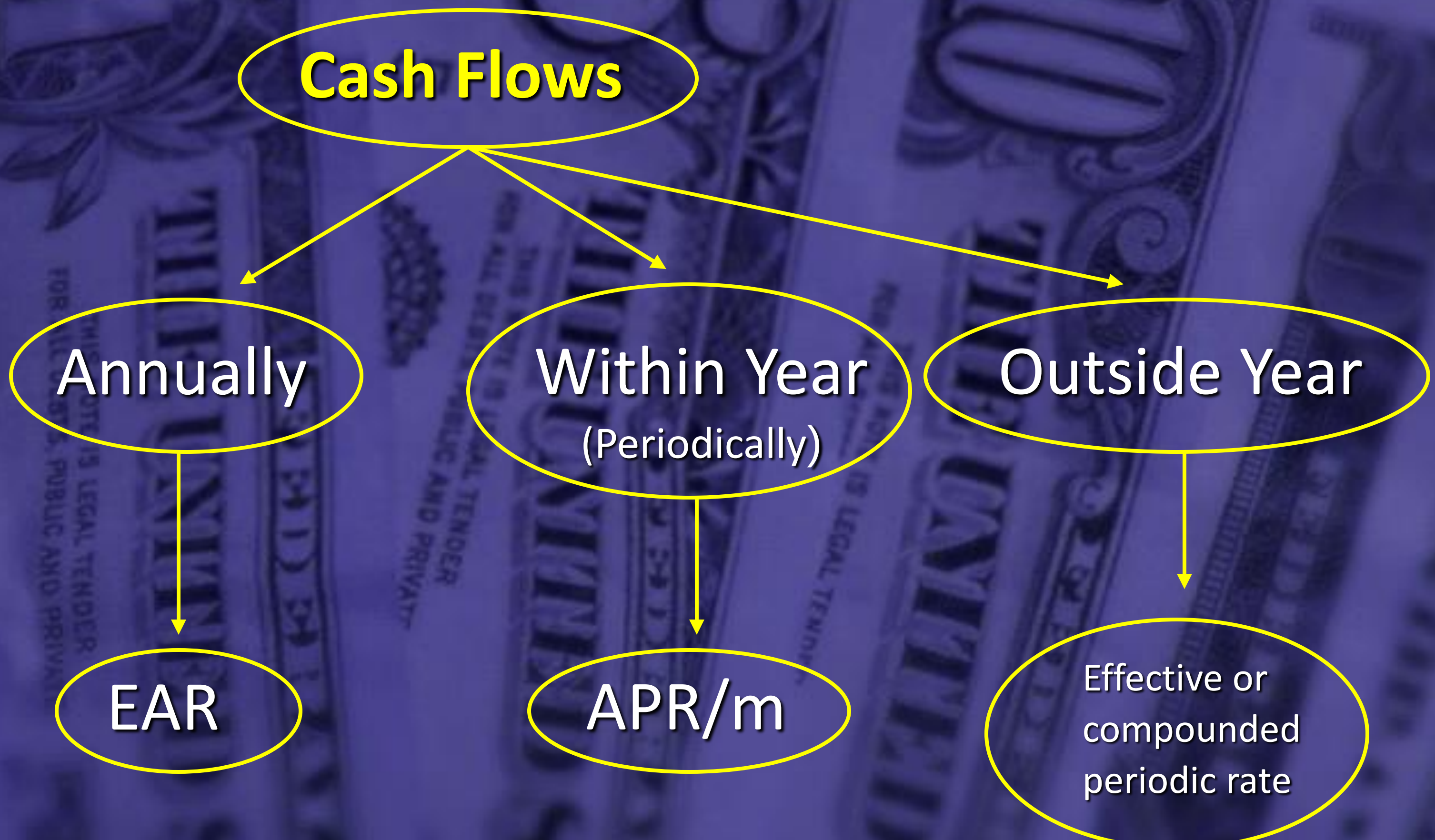
ANSWER = \$433,242

Periodic Interest Rates

- Is receiving 12% per year equivalent to 1% per month?
  - No, since multiplying 1% per month by 12 ignores compounding
  - At 1% per month, $1 invested at the beginning of the year would be worth $(1.01)^{12}$ or $1.126825 at the end of the year.
  - Effective annual rate (EAR) is 12.6825%
  - Nominal interest rate is 12% compounded monthly (APR)
- General rule: r% compounded m times per year
  - $EAR = [1 + (r/m)]^m - 1$
  - $EAR = [1+(0.12/12)]^{12} - 1 = 0.126825 = 12.6825\%$

What interest rate is used in PV calculations?

– **RULE: Maintain Cash Flow Frequency and adjust interest rate accordingly**



Inflation

- Note – Present value is measured at a given point in time (t=0), therefore inflation does not matter!
- Be consistent: Rule:
  - Use nominal interest or discount rates for nominal cash flows (most common case!)
  - Use inflation-adjusted (real) discount rates for inflation-adjusted cash flows
- Nominal rate (R) is based on change in $
- Real rate (r) is based on change in purchasing power

Inflation

- If h denotes the inflation rate, then:
- $1 + R = (1 + r) \times (1 + h)$
- $R = (r + h) + (r \times h)$
- $r \times h$ is often small and dropped:
- $R \approx (r + h)$

- *Example: If investors require a 10% real rate of return and h=3%, what is their required nominal rate?*
- $1 + R = (1.1) \times (1.03) \Rightarrow R = 13.3\%$
- $R \approx 10\% + 3\% \approx 13\%$

Example 1

- Value the following stream of cash flows: In year 5 ($t=5$) you receive \$1,000, followed by 20 yearly payments of \$5,000. The APR is 8% compounded monthly.
- *Annual Cash flows: Need EAR with m=12* → *EAR =* $[1 + (0.08 / 12)]^{12} - 1 = 8.3\%$
- *Single cash flow in year 5 + Annuity of 20 cash flows starting in year 6 (answer in year 5)*
- *PV =* $\$1{,}000 / 1.083^5 + [1 / 1.083^5] \times [\$5{,}000 / 0.083] \times [1 - (1 / 1.083^{20})]$
- *PV = \$32,898.48*

Example 2

- Value the following stream of cash flows: You will receive monthly payments of $500 starting exactly one year from now in perpetuity. The APR=18%.
- *Monthly cash flows so we need a monthly rate with m=12 → r = 18%/12 = 1.5%*
- *Perpetuity of monthly cash flows starting at t=12 (year 1 = 12 months)*
- *PV =* $[1 / 1.015^{11}] \times [\$500 / 0.015]$
- *PV = $28,297.77*

Example 3

- Value the following stream of cash flows: You will receive 50 semi-annual payments of $100 starting one half year from now. The APR is 12% compounded monthly.

- *Cash flows are semi-annual, so we need a semi-annual rate with m=2. However, within each half year, there is still compounding (monthly), so we need an effective semi-annual rate, with m=6 or 6 compounding periods per half year.*

- *$r = 12\%/2 = 6\%$ → Turn into effective using $r = [1 + (0.06 / 6)^6] – 1 = 6.152\%$ [Alternatively: $r = [1 + (0.12 / 12)]^6 – 1 = 6.152\%$]*

- *This is an annuity starting at t=1 (the first half year) with t=50*

- *$PV = [\$100 / 0.06152] \times [1 – (1 / 1.06152^{50})]$ → $PV = \$1,543.34$*

Example 4

- Value the following stream of expected annual cash flows. Three years from now, the expected cash flow is \$2.50, the next year \$2.75, growing thereafter at 12% per year for the following 18 years, after which cash flows will continue forever, but no further growth. Use a 9% discount rate.

- CF_3 = \$2.50
- CF_4=\$2.75
- CF_5= \$2.75 × (1.12) *(first year of growth)*
- CF_6=\$2.75 × $(1.12)^2$ *(second year of growth)*
- . . .
- CF_{22}=\$2.75 × $(1.12)^{18}$ *(final (18th) year of growth)*
- CF_{23} = CF22 *(beginning of a non-growing perpetuity)*
- PV = $(2.50 / 1.09^3) + (1/1.09^3) \times (2.75 / (0.09-0.12) \times (1 - ((1.12/1.09)^{19})$
  $+ (1/1.09^{22}) \times (21.15 / 0.09) = \$85.00$